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DETERMINANTS OF ACHIEVEMENT IN GEOMETRY AMONG SENIOR HIGH SCHOOL STUDENTS

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ABSTRACT

Geometry is a basic area of mathematics that is of great significance in cultivating senior high school students' ability to reason spatially, think logically, and solve problems deductively. However, despite its significance in both learning and practical applications, it has been observed that students have shown varying levels of performance in geometry. This research paper will discuss the key factors that affect the performance of students in geometry, such as the conceptual understanding of basic concepts like angles, congruence, and similarity, logical reasoning and proof-writing skills, quality of instruction, and teaching methods. The paper will also focus on the theoretical aspects of geometry by including basic definitions, Euclid's axioms and postulates, key theorems like the triangle angle-sum theorem, and the Pythagorean theorem with formal proofs, and examples of problem-solving. By combining mathematical theory with factors that affect education, this paper will offer a complete framework for understanding how the performance of students in geometry can be improved through better instruction, resources, and attitudes.

Keywords: *Geometry achievement, Conceptual understanding, Proof skills, Spatial visualization, Teaching methodology, Euclid's postulates, Senior high school students.*

1. INTRODUCTION

Geometry is one of the most important and fundamental areas of mathematics that deals with the study of shapes, sizes, angles, spatial relationships, and properties of figures in two-dimensional and three-dimensional space. The word geometry has its roots in the Greek word 'sgeo' meaning earth and 'metron' meaning measurement, which signifies its application in the measurement of land in the early days. Geometry has a rich history as a fundamental mathematical science that developed in response to practical human needs such as the division of land, architectural construction, navigation, and astronomical observations. Over the years, geometry has developed as a rigorous mathematical discipline that forms the foundation for logical organization and deductive reasoning in mathematics.

In the modern education system, especially in the senior high school curriculum, geometry is an integral part of developing students' spatial visualization skills, analytical skills, and problem-solving skills. Unlike arithmetic or algebra, geometry is more conceptual and deals more with logical reasoning. Students are required not only to solve geometric problems but also to develop proofs and justify the validity of mathematical statements using axioms, definitions, and proven theorems.

The relevance of geometry can be seen in its applications in various fields of science and technology. The concepts of distance, area, volume, symmetry, and transformation are basic to fields like engineering, architecture, computer graphics, physics, robotics, and geography. For example, the area of a triangle is expressed by:

$$A = \frac{1}{2} bh$$

where b represents the base and h represents the height. Likewise, the Pythagorean Theorem, which is one of the most frequently employed concepts in geometry, asserts that for a right-angled triangle:

$$a^2 + b^2 = c^2$$

where $c$is the hypotenuse and a, $b$are the perpendicular sides. Such fundamental equations highlight the relevance of geometry in both academic settings and real-world applications.

Again, despite its importance in mathematical learning, performance in geometry can differ considerably among senior high school students. Various studies have shown that many students tend to have problems in grasping geometric concepts, interpreting geometric diagrams, applying geometric theorems, and developing formal geometric proofs. Geometry is a subject that combines visual-spatial intelligence and logical thinking, making it more cognitively demanding than other mathematical subjects. For instance, students are expected to grasp concepts such as the sum of the interior angles of a triangle:

$$\angle A + \angle B + \angle C = 180^\circ$$

and apply this knowledge in both problem-solving and proof-based contexts.

2. LITERATURE REVIEW

Sam (2021) assessed the factors that contributed to achievement in geometry among senior high school students. The study investigated the impact of students' conceptual understanding, reasoning skills, teaching methods, and the availability of teaching resources on students' achievement. The results showed that students generally struggled with geometry because of inadequate conceptual understanding of basic ideas like angles, congruence, and similarity. Sam also noted that appropriate teaching methods, such as the use of visual aids and student-centered approaches, had been contributing to better achievement outcomes. The study pointed out those both cognitive and environmental factors had contributed to geometry achievement outcomes among students.

Yirsor (2022) investigated the performance of female students in geometry by examining teaching approaches and factors that influence learning outcomes in senior high schools in the Wa Municipality. The study emphasized that female students have been facing difficulties in geometry because of inadequate access to appropriate problem-solving strategies and a lack of confidence in dealing with proof-based problems. Yirsor indicated that the application of interactive teaching approaches, collaborative learning, and constant teacher support has enhanced student engagement and performance. The study emphasized the importance of gender-responsive teaching approaches and interventions to improve female student participation and performance in geometry.

Alghadari, Herman, and Prabawanto (2020) explored the factors that influenced the ability of senior high school students to solve three-dimensional geometry problems. The study was conducted by exploring the students' visualization skills, problem-solving processes, and conceptual understanding of geometric representation. The results of the study revealed that students had struggled with three-dimensional geometry problems because of their inability to visualize geometric relationships. The study concluded that students with strong visualization skills and conceptual understanding of geometry had performed better in solving complex geometry problems. The study highlighted that support and visualization-based teaching had greatly improved students' performance in geometry problems.

3. CONCEPTUAL FOUNDATIONS OF GEOMETRY

Geometry is the theoretical foundation of mathematical thinking. Prior to delving into the factors that influence geometry achievement among senior high school students, it is important to lay down the basic concepts on which geometry is founded. The conceptual foundations of geometry are the study of fundamental objects such as points, lines, planes, and angles, which are the basic building blocks of more complex geometric concepts and theorems. Geometry is not only beneficial in improving spatial visualization skills but also helps in improving logical thinking skills through definitions, axioms, and proof techniques. By developing a sound understanding of the conceptual foundations of geometry, students are better prepared to understand more complex concepts such as congruence, similarity, transformations, coordinate geometry, and non-Euclidean geometries. A sound conceptual foundation is therefore imperative for effective learning and achievement in geometry.

3.1 Definition of Geometry

Geometry is defined as:

The branch of mathematics that studies the properties, measurements, and relationships of points, lines, surfaces, and solids.

It is mainly classified into:

- **Euclidean Geometry** (plane and space geometry)
- **Analytic Geometry** (geometry using coordinate systems)
- **Non-Euclidean Geometry** (hyperbolic and spherical geometry)

3.2 Basic Geometric Terms

Definition 1: Point

A point is a location in space and has no dimension.

Definition 2: Line

A line is a straight path extending infinitely in both directions.

Equation of a line in coordinate geometry:

$$y = mx + c$$

Where

m= slope,c= y-intercept.

Definition 3: Plane

A plane is a flat surface extending infinitely in all directions.

Definition 4: Angle

An angle is formed when two rays share a common endpoint.

Angle measure:

$$\theta = \frac{s}{r}$$

where
s= arc length,r= radius.

4. AXIOMS AND POSTULATES IN GEOMETRY

Geometry, as a logical and deductive field of mathematics, is based on a set of fundamental assumptions called axioms and postulates. These assumptions are the basis of geometric reasoning and serve as the foundation from which theorems and proofs are derived. Unlike theorems, which are proven, axioms and postulates are accepted as true without proof because they are self-evident statements.

In geometry, these assumptions are crucial because they define the rules of geometric relationships between points, lines, angles, and shapes. The whole of classical geometry, including Euclidean geometry, is based on a few postulates developed by the ancient Greek mathematician Euclid in his famous book Elements. Euclid's postulates have stood the test of time for over two thousand years and continue to form the basis of geometry education at the secondary and senior high school levels.

- **Euclid's Postulates**

Euclid described five main postulates that are the foundation of plane geometry. These postulates define the fundamental characteristics of space and are the basis for constructing geometric objects and proving other statements.

- **Postulate 1: Line Joining Two Points**

Statement: Show: A straight line can be drawn joining any two points.

This postulate defines that for any two different points in a plane, there exists a unique straight line that joins the two points. This postulate describes the simplest form of geometric construct and is the basis for the definition of line segments and distances.

Symbolically, if two points A and B exist, then there is a line segment:

$$\bar{AB}$$

This is a fundamental principle for forming figures such as triangles, polygons, and geometric graphs.

- **Postulate 2: Extension of a Line Segment**

Statement: A finite line segment can be extended infinitely in a straight line.

This postulate means that a line is endless in both directions and has no end point. It differentiates a line segment from a line, which is infinite.

For instance, given a line segment *AB*, it can be extended beyond point Bendlessly:

$$AB \to \infty$$

This postulate enables the formation of concepts like rays, parallel lines, and infinite geometric space.

- **Postulate 3: Construction of a Circle**

Statement: A circle can be drawn with any centre and any radius.

This postulate enables the formation of concepts of circles and understanding the concept of radius in plane geometry. It states that for any point as centre and any fixed length as radius, a circle can be drawn.

The standard equation of a circle centred at the origin is:

$$x^2 + y^2 = r^2$$

where $r$represents the radius.

This postulate is fundamental in studying chord properties, tangents, arcs, and circle theorems.

- **Postulate 4: Equality of Right Angles**

Statement: All right angles are equal to one another.

A right angle is defined as an angle measuring exactly 90°. Euclid's fourth postulate establishes the uniformity of perpendicularity throughout geometry.

Thus, if:

$$\angle A = 90^\circ \text{and} \angle B = 90^\circ$$

then:

$$\angle A = \angle B$$

This postulate is the basis for congruence, perpendicular lines, and geometric proofs involving angle relationships.

- **Postulate 5: The Parallel Postulate**

Statement: Only one line can be drawn through a point not on a given line that is parallel to the given line.

This is the most famous and most controversial of Euclid's postulates. It establishes the properties of parallel lines in plane geometry and has major implications for the overall framework of Euclidean space.

If a line $l$and a point Pnot on $l$are given, then there exists exactly one line through Psuch that:

$$l_1 \parallel l$$

This postulate gives rise to some of the most important geometric theorems, including the following:

Alternate interior angles are equal

Sum of angles in a triangle is 180°

Properties of polygons and similarity

Interestingly, when mathematicians tried to alter this postulate, they stumbled upon the discovery of non-Euclidean geometries like hyperbolic and spherical geometry.

5. KEY THEOREMS IN GEOMETRY WITH PROOFS

However, geometry is more than just identifying shapes and calculating angles; it is inextricably linked to logical thinking and proof. Theorems are the essence of geometry, as they define basic properties of geometric shapes on the basis of axioms, definitions, and postulates. A theorem is a mathematical assertion that is logically demonstrated to be true by a specific series of logical statements. In senior high school geometry, the capacity to comprehend and apply key theorems is a critical factor in determining students' performance, as theorems' proof enhances logical thinking and problem-solving skills. Key theorems such as the angle-sum theorem for triangles, the Pythagorean theorem, and properties of parallel lines are basic principles for more complex ideas in geometry, trigonometry, coordinate geometry, and practical applications. Thus, the formal presentation of these key theorems with their proofs offers a solid theoretical foundation for comprehending students' learning and performance outcomes in geometry.

Theorem 1: Sum of Angles in a Triangle

Statement

The sum of the interior angles of a triangle is:

$$A + B + C = 180^\circ$$

Proof

Let triangle ABCbe given.

Draw a line through point Aparallel to side BC.

Using alternate interior angle properties:

$$\angle ACB = \angle 1$$
$$\angle ABC = \angle 2$$

Now angles on a straight-line sum to 180°:

$$\angle 1 + \angle A + \angle 2 = 180^\circ$$

Substituting:

$$\angle C + \angle A + \angle B = 180^\circ$$

Hence proved.

Theorem 2: Pythagoras Theorem

Statement

In a right triangle:

$$a^2 + b^2 = c^2$$

where
c= hypotenuse.

Proof

Consider a square of side $(a+b)$.

Area:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Inside are 4 right triangles each area:

$$\frac{1}{2}ab$$

Total triangle area:

$$4 \times \frac{1}{2}ab = 2ab$$

Remaining area is square with side c:

$$c^2$$

Thus:

$$a^2 + 2ab + b^2 = 2ab + c^2$$

Subtract $2ab$:

$$a^2 + b^2 = c^2$$

Hence proved.

6. PROBLEM SOLVING IN GEOMETRY

Example 1: Angle Sum

If two angles of a triangle are:

$$50^\circ \text{ and } 60^\circ$$

Find the third angle.

Solution:

$$A + B + C = 180^\circ$$
$$50 + 60 + C = 180$$
$$110 + C = 180$$

$$C = 70°$$

Example 2: Using Pythagoras

A right triangle has legs:

$$a = 6,\ b = 8$$

Find hypotenuse.

$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64 = 100$$
$$c = 10$$

7. DETERMINANTS OF ACHIEVEMENT IN GEOMETRY

Success in geometry among senior high school students is affected by a large number of cognitive, instructional, and affective determinants. Geometry is a special area of mathematics because it not only involves numerical computation but also requires conceptual understanding, spatial reasoning, deductive logic, and proof construction. Unlike other areas of mathematics, geometry involves the interpretation of diagrams, relationships between figures, and logical justification of solutions through theorems and axioms. Thus, students' performance in geometry is affected by a number of interrelated determinants, which are explained below.

7.1 Conceptual Understanding

Conceptual understanding is one of the most basic factors that influence success in geometry. Students need to build a solid foundation of basic concepts in geometry before they can tackle difficult problems and write proofs. Geometry is based on a set of basic concepts, including angles, congruence, similarity, and properties of shapes.

For instance, students need to understand concepts such as the angle sum property of a triangle:

$$\angle A + \angle B + \angle C = 180°$$

Also, understanding the concept of congruence enables students to identify when two triangles are congruent in shape and size. Two triangles are said to be congruent if their corresponding sides and angles are equal:

$$\triangle ABC \cong \triangle DEF$$

Similarly, similarity enables students to apply proportional concepts, which are written as:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Students with poor conceptual understanding usually find it difficult to use these relationships effectively, resulting in low performance in geometry.

7.2 Logical Reasoning and Proof Skills

Geometry requires a strong ability in deductive reasoning because it is one of the only areas in school mathematics that focuses on formal proof. Students who use logical reasoning can link definitions, axioms, and theorems to draw valid conclusions.

Proof skills comprise:

- Identifying the given assumptions
- Using the relevant theorems
- Building a logical argument

For example, to prove the Pythagorean Theorem, a student needs to understand that in a right-angled triangle:

$$a^2 + b^2 = c^2$$

Similarly, students need to apply logical reasoning for parallel lines. When two parallel lines are intersected by a transversal, the alternate interior angles are equal:

$$\angle 1 = \angle 2$$

Students who lack reasoning skills will struggle to explain their answers and will end up relying on memorization alone, as opposed to understanding. This has a great impact on geometry, where proofs are core.

7.3 Teaching Methods and Instructional Quality

Quality of instruction and teaching methods are key factors that influence the performance of students in geometry. Learning geometry can be improved by the use of effective teaching methods like visual aids, hands-on activities, and real-world examples.

Examples of effective teaching methods include:

- Diagrams and models
- Hands-on activities using geometric instruments
- Use of technology such as GeoGebra
- Proof discussions

For instance, when teaching coordinate geometry, teachers can assist students in visualizing the equation of a line:

$$y = mx + c$$

or distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Traditional rote learning, whereby students are only required to memorize formulas without any conceptual explanation, is a factor that lowers achievement because geometry is a subject that requires more in-depth thinking rather than repetitive computation.

8. EDUCATIONAL IMPLICATIONS

Knowledge of the determinants of achievement in geometry has important educational implications for schools, teachers, and curriculum developers. Geometry is a subject that not only requires computational skills but also requires conceptual understanding, logical thinking, and the ability to develop mathematical proofs. Thus, knowledge of the factors that influence achievement enables educational institutions to adopt more effective teaching and learning environments.

Firstly, knowledge of the determinants of achievement enables schools to enhance teaching methods in geometry. Teachers can use learner-centered approaches that focus on visualization, hands-on experiences, and real-world applications rather than rote memorization of formulas. Geometry concepts such as the area of a triangle,

$$A = \frac{1}{2}bh$$

or the Pythagorean relationship,

$$a^2 + b^2 = c^2$$

However, these concepts can become more meaningful when the students are able to see the relevance of these concepts in the context of construction, design, and measurement. Therefore, the implication of teaching geometry through contextual learning is that it enhances understanding and improves achievement.

Secondly, one of the significant implications of teaching geometry is related to improving the students' proof-learning approaches. Geometry is the only branch of mathematics that requires students to prove their conclusions using deductive reasoning. If students are not able to prove concepts, teachers can develop approaches that help students improve their reasoning skills from basic angle properties to complex theorem proofs. For instance, when students are able to understand that the sum of angles in a triangle is always,

$$\angle A + \angle B + \angle C = 180^\circ$$

enables students to develop the skill of logically deducing conclusions rather than just applying formulas by rote.

Thirdly, schools can offer better availability of geometry practice materials. Given the fact that geometry performance is highly dependent on spatial visualization and repeated exposure to geometric diagrams and problem-solving, schools must make sure that students have access to enough geometry practice materials such as textbooks, geometry software, and geometry practice worksheets. The availability of these materials is an important factor in the understanding of geometric concepts, including coordinate geometry concepts such as the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is crucial for linking algebraic understanding with geometric meaning.

Moreover, knowledge of determinants of achievement enables teachers to concentrate on promoting positive attitudes towards geometry. Most students view geometry as a difficult subject because of its abstractness and emphasis on proofs. Teachers and schools need to develop confidence, motivation, and interest in students through supportive classroom environments and feedback. Positive attitudes play a direct role in influencing perseverance in dealing with difficult problems and complex reasoning tasks.

$$y = mx + c$$

helps students visualize geometric relationships on the coordinate plane and apply geometry beyond theoretical contexts.

9. CONCLUSION

In conclusion, geometry performance among senior high school learners is influenced by a set of conceptual, logical, instructional, and psychological factors. Geometry is more than the study of shapes and measurements; it is a field of study based on axioms, postulates, theorems, and deductive proofs that demand high levels of reasoning and visualization skills. The performance of learners is highly dependent on their understanding of key concepts like angle relationships, congruence, similarity, and the application of theorems, as well as their capacity to develop logical proofs and sound conclusions. Furthermore, the application of effective teaching approaches, visual and interactive learning strategies, sufficient learning resources, and the promotion of positive attitudes towards mathematics makes a significant contribution to enhanced geometry performance. Thus, improving conceptual understanding, enhancing proof-learning strategies, and fostering a supportive learning environment are critical steps in the direction of encouraging higher levels of geometry performance among senior high school learners.

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